Triaxiality-related nuclear phenomena in the A≈100 mass region

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Triaxial nuclear shape



Triaxial shape: no single characteristic quantity, like Q_o for quadrupole deformation

Phenomena related with triaxial shape:

- gamma bands

(low-spin region)

- wobbling motion

- chiral rotation



(medium-spin region)

(medium-spin region)



Subject of the present talk: our recent results on nuclear wobbling motion and chirality in the A~100 mass region

Radu Budaca: nice talk on the theoretical aspects (Tuesday afternoon)

Wobbling motion of the triaxial nucleus

A. Bohr, B.R. Mottelson, Nuclear Structure, vol. II (Benjamin, New York, 1975)

In even-even triaxially deformed nuclei:

 $H_{\rm rot} = A_3 \hat{\mathbf{J}}_3^2 + A_1 \hat{\mathbf{J}}_1^2 + A_2 \hat{\mathbf{J}}_2^2 \qquad A_i = \frac{\hbar^2}{2\mathcal{J}_i}$ If

 $A_1 > A_2 > A_3 (\mathcal{J}_1 < \mathcal{J}_2 < \mathcal{J}_3)$

then slightly above the yrast line, the energy spectrum is:

 $H = A_3 I(I+1) + (n+\frac{1}{2})\hbar\omega_w$

"The quantum number n describes the precession motion of the axes with respect to the direction of J; for small amplitudes, this motion has the character of a harmonic vibration with frequency w_w ."

The "wobbling frequency" is:

 $\hbar\omega_w = 2I\sqrt{(A_1 - A_3)(A_2 - A_3)}$

thus, it is increasing with increasing angular momentum.



Expected experimental characteristics



ΔI=2 E2

- $\Delta I=1 M1+E2$ with
- dominant E2 character
- A series of $\Delta I=2$ E2 bands
- Consecutive bands have different signature
- ΔI=1 M1+E2 transitions between the consecutive bands with dominant E2 character
- $E_w(I)$ is increasing with spin

 $\hbar \omega_{W} \approx E_{W}(I) = E(I, n=1) - [E(I-1, n=0) + E(I+1, n=0)]/2$

Experimentally observed wobbling in nuclei



Theoretical understanding of the observed features of wobbling in odd-A nuclei

In spite of the limited number of the experimental observations of nuclear wobbling, many theoretical papers have been published to describe the observed features in odd-A nuclei.

They use different formalisms and different assumptions for the moments of inertia.

- S. Frauendorf and F. Dönau, Phys. Rev. C 89, 014322 (2014)
- Q. B. Chen *et* al., Phys. Rev. C 90, 044306 (2014)
- S. Frauendorf and F. Dönau, Phys. Rev. C 92, 064306 (2015).
- Q. B. Chen et al., Phys. Rev. C 94, 044301 (2016)
- Q. B. Chen et al., Phys. Rev. C 94, 054308 (2016)
- K. Tanabe and K. Sugawara-Tanabe, Phys. Rev. C 95, 064315 (2017).
- A. A. Raduta et al., Phys. Rev. C 96, 054320 (2017)
- M. Shimada *et* al., Phys. Rev. C 97, 024318 (2018)
- R. Budaca, Phys. Rev. C 97, 024302 (2018)

It provides an approximation, which interprets the observed behavior of $E_w(I)$.

It also leads to the classification of the wobbling cases as:

- simple wobbling (even-even nuclei)
- **longitudinal wobbling** (odd-A nuclei) The AM of the odd particle is aligned parallel with the axis of the largest MO.
- **transverse wobbling** (odd-A nuclei) The AM of the odd particle is aligned perpendicular to the axis of the largest MO.
- **simple wobbling**: wobbling energy is increasing with increasing spin
- longitudinal wobbling: wobbling energy is increasing with increasing spin
- transverse wobbling: wobbling energy is decreasing with increasing spin

Where to search for new wobbling bands?



- The region around ¹⁰⁸Ru shows the strongest deviation from axial symmetry at low energy
- No wobbling has been observed before our studies
- Bands based on particle-type high_j **neutron** configuration exist

Search for wobbling bands in ¹⁰⁵Pd



Search for wobbling bands in ¹⁰⁵Pd



CDFT calculations for ¹⁰⁵Pd

Constrained triaxial covariant density functional theory (CDFT) calculations were performed for the low-energy configurations.

PHYSICAL REVIEW C 73, 037303 (2006)

Possible existence of multiple chiral doublets in ¹⁰⁶Rh

J. Meng,^{1,2,3,*} J. Peng,¹ S. Q. Zhang,¹ and S.-G. Zhou^{2,3} ¹School of Physics, Peking University, Beijing 100871, China ²Institute of Theoretical Physics, Chinese Academy of Science, Beijing 100080, China ³Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China (Received 30 March 2005; published 15 March 2006)

Adiabatic and configuration-fixed constrained triaxial relativistic mean field (RMF) approaches are developed for the first time. A new phenomenon, the existence of multiple chiral doublets ($M\chi D$), i.e., more than one pair of chiral doublet bands in one single nucleus, is suggested for ¹⁰⁶Rh based on the triaxial deformations and their corresponding proton and neutron configurations.

Triaxial shape has been found for the $v(1h_{11/2})$ configuration.



Ç	State	$E_{\mathbf{x}}$	(eta,γ)	Valence configuration	Unpaired configuration	π
	Α	0.00	$(0.19, \ 0.0^{\circ})$	$\pi(1g_{9/2})^6\otimes u(1g_{7/2})^5(2d_{5/2})^4$	$ u(1g_{7/2})^1$	+
	В	0.38	$(0.23, 22.2^{\circ})$	$\pi(1g_{9/2})^8\otimes u(1g_{7/2})^5(2d_{5/2})^4$	$ u(1g_{7/2})^1$	+
	С	0.51	$(0.27, 24.9^{\circ})$	$\pi(1g_{9/2})^8 \otimes \nu(1g_{7/2})^6 (2d_{3/2})^2 (1h_{11/2})^1$	$ u(1h_{11/2})^1$	_
	D	0.73	$(0.29, 30.7^{\circ})$	$\pi(1g_{9/2})^8 \otimes \nu(1g_{7/2})^3 (2d_{5/2})^2 (2d_{3/2})^2 (1h_{11/2})^2$	$ u(1g_{7/2})^1$	+

PRM calculations for ¹⁰⁵Pd



Particle rotor model (PRM) calculations were performed for the $v(1h_{11/2})$ configuration.

- the results describe well the experimental energies of the wobbling bands
- they also describe well the experimental properties of the gamma decays

These results prove the wobbling nature of the observed Band C.

This is the first observed wobbling band in this mass region, and the first wobbling band based on neutron configuration.

		δ		$[B(M1)_{\rm out}/B(E2)_{\rm in}]$	$](\mu_N^2/e^2b^2)$	$[B(E2)_{\rm out}/B(E2)_{\rm in}]$	
$I_i^{\pi} \to I_f^{\pi}$	E_{γ} (keV)	Expt	PRM	Expt	PRM	Expt	PRM
$17/2^- \rightarrow 15/2^-$	991	1.8 ± 0.5	2.38	0.162 ± 0.097	0.105	0.66 ± 0.18	0.736
$21/2^- \rightarrow 19/2^-$	1034	2.3 ± 0.3	2.30	0.089 ± 0.026	0.069	0.60 ± 0.09	0.465
$25/2^- \rightarrow 23/2^-$	994	2.7 ± 0.6	1.99	0.029 ± 0.016	0.057	0.34 ± 0.07	0.329

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Search for n=2 wobbling bands in ¹⁰⁵Pd



They decay mainly to the yrast band

Not n=2 wobbling bands

Plans for further search in the A~100 mass region:

- search for n=2 wobbling band
- search for longitudinal wobbling
- search for wobbling band with hole-type high-J proton configuration

Nuclear chirality

NUCLEAR PHYSICS A



Nuclear Physics A 617 (1997) 131-147

Tilted rotation of triaxial nuclei

S. Frauendorf, Jie Meng¹ Institut für Kern- und Hadronenphysik, Forschungszentrum Rossendorf e.V., PF 510119, 01314 Dresden, Germany

Received 14 November 1996





Another recent review:

K Starosta and T Koike, Phys. Scr. 92 (2017) 093002

They do not represent static chirality

- D. Tonev et al., Phys. Rev. Lett. 96, 052501 (2006)

- C. M. Petrache et al., Phys. Rev. Lett. 96, 112502 (2006)

Multiple chiral doublet bands (M χ D)

PHYSICAL REVIEW C 73, 037303 (2006)

Possible existence of multiple chiral doublets in ¹⁰⁶Rh

J. Meng,^{1,2,3,*} J. Peng,¹ S. Q. Zhang,¹ and S.-G. Zhou^{2,3} ¹School of Physics, Peking University, Beijing 100871, China ²Institute of Theoretical Physics, Chinese Academy of Science, Beijing 100080, China ³Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China (Received 30 March 2005; published 15 March 2006)

Adiabatic and configuration-fixed constrained triaxial relativistic mean field (RMF) approaches are developed for the first time. A new phenomenon, the existence of multiple chiral doublets (M χ D), i.e., more than one pair of chiral doublet bands in one single nucleus, is suggested for ¹⁰⁶Rh based on the triaxial deformations and their corresponding proton and neutron configurations.



Evidence for Multiple Chiral Doublet Bands in ¹³³Ce

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Two distinct sets of chiral-partner bands have been identified in the nucleus ¹³³Ce. They constitute a multiple chiral doublet, a phenomenon predicted by relativistic mean field (RMF) calculations and observed experimentally here for the first time. The properties of these chiral bands are in good agreement with results of calculations based on a combination of the constrained triaxial RMF theory and the particle-rotor model.



First hints for multiple chiral doublet bands



Multiple chiral doublets in ¹⁰³Rh

Experiment: with the GAMMASPHERE array ⁹⁶Zr(¹¹B,4n)¹⁰³Rh fusion-evaporation reaction Beam energy of 40 MeV ~ 9x10⁸ quadruple- and higher-fold events Strongest reaction channel

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Multiple chiral doublets in ¹⁰³Rh

Calculations in 3 steps (J. Meng et al., PRC 73, 037303 (2006))

- adiabatic and configuration fixed constrained CDFT: configurations, deformations



A,B,C,D,E: configurations with no unpaired neutron a,b,c,d,e: configurations with unpaired neutrons

a:
$$\pi(1g_{9/2})^{-1} \otimes \nu(1h_{11/2})^{1}(2d_{5/2})^{1}$$

C:
$$\pi(1g_{9/2})^{-1} \otimes \nu(1h_{11/2})^{1}(1g_{7/2})^{-1}$$

d:
$$\pi(1g_{9/2})^{-1} \otimes \nu(1h_{11/2})^{1}(1h_{11/2})^{1}$$

- TAC CDFT: rotational behavior of bands (only the high-J orbitals fixed)



Positive parity bands: $\pi(1g_{9/2})^{-1} \otimes \nu(1h_{11/2})^{2}$

Negative parity bands: $\pi(1g_{9/2})^{-1} \otimes \nu(1h_{11/2})^{1}$

Effective configuration for the negative parity bands: $\pi(1g_{9/2})^{-1} \otimes \nu(1h_{11/2})^{1}(1g_{7/2})^{1}$

Multiple chiral doublets in ¹⁰³Rh

- PRM calculations: energy spectra and B(M1)/B(E2) ratios

 β and γ were taken from the TAC CDFT results moments of inertia were fitted to the energy spectra of the bands



Possible multiple chiral doublets in ¹⁰⁴Rh

PHYSICAL REVIEW C 77, 024309 (2008) **Search for multiple chiral doublets in rhodium isotopes** J. Peng,^{1,2,3} H. Sagawa,² S. Q. Zhang,^{3,4,*} J. M. Yao,³ Y. Zhang,³ and J. Meng^{3,4,5,6} ¹Department of Physics, Beijing Normal University, Beijing 100875, People's Republic of China ²Center for Mathematical Sciences, University of Aizu, Aizu-Wakamatsu, 965-8580 Fukushima, Japan ³School of Physics, and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, People's Republic of China ⁴Institute of Theoretical Physics, Chinese Academy of Science, Beijing 100080, People's Republic of China ⁵Department of Physics, University of Stellenbosch, Stellenbosch, South Africa ⁶Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, People's Republic of China

(Received 5 December 2007; published 25 February 2008)



State	${E}_{ m cal}$	$eta_{ ext{cal}}$	$\gamma_{\rm cal}$	Valence nucleon configuration
А	-888.51	0.23	20.9°	$\pi g_{9/2}^{-3} \otimes \nu (2d_{5/2})^1$
В	-888.40	0.26	23.4°	$\pi g_{9/2}^{-3} \otimes \nu h_{11/2}^1 \blacktriangleleft$
C	-887.84	0.28	20.6°	$\pi g_{9/2}^{-3} \otimes \nu [h_{11/2}^2 g_{7/2}^{-1}]$
D	-887.18	0.34	9.4°	$\pi g_{7/2}^1 \otimes \nu h_{11/2}^3$
Е	-886.69	0.39	4.7°	$\pi (2p_{3/2})^{-1} \otimes \nu h_{11/2}^3$
F	-885.05	0.44	2.9°	$\pi (2p_{3/2})^{-1} \otimes \nu (h_{11/2}^4 g_{9/2}^{-1})$
G	-885.90	0.18	0.1°	$\pi g_{9/2}^{-5} \otimes u (2d_{5/2})^1$
	A B C D E F G	$\begin{array}{c c} {\rm State} & E_{\rm cal} \\ \hline {\rm A} & -888.51 \\ \hline {\rm B} & -888.40 \\ {\rm C} & -887.84 \\ \hline {\rm D} & -887.18 \\ {\rm E} & -886.69 \\ {\rm F} & -885.05 \\ {\rm G} & -885.90 \\ \hline \end{array}$	$\begin{array}{c cccc} {\rm State} & E_{\rm cal} & \beta_{\rm cal} \\ \hline {\rm A} & -888.51 & 0.23 \\ \hline {\rm B} & -888.40 & 0.26 \\ \hline {\rm C} & -887.84 & 0.28 \\ \hline {\rm D} & -887.18 & 0.34 \\ \hline {\rm E} & -886.69 & 0.39 \\ \hline {\rm F} & -885.05 & 0.44 \\ \hline {\rm G} & -885.90 & 0.18 \\ \hline \end{array}$	$\begin{array}{c ccccc} {\rm State} & E_{\rm cal} & \beta_{\rm cal} & \gamma_{\rm cal} \\ \hline {\rm A} & -888.51 & 0.23 & 20.9^\circ \\ \hline {\rm B} & -888.40 & 0.26 & 23.4^\circ \\ {\rm C} & -887.84 & 0.28 & 20.6^\circ \\ \hline {\rm D} & -887.18 & 0.34 & 9.4^\circ \\ {\rm E} & -886.69 & 0.39 & 4.7^\circ \\ {\rm F} & -885.05 & 0.44 & 2.9^\circ \\ {\rm G} & -885.90 & 0.18 & 0.1^\circ \\ \end{array}$

The positive-parity band structure is not well known.



Possible multiple chiral doublets in ¹⁰⁴Rh (PRELIMINARY)

Experiment: with GAMMASPHERE array ¹¹B beam on ⁹⁶Zr target at 40 MeV fusion-evaporation reaction ~ 9x10⁸ quadruple- and higher-fold events second strongest reaction channel

- Based on their parity and alignment, bands 1,2,3,4,5 have the same $\pi(1g_{9/2})^{-1} \otimes \nu(1h_{11/2})^2$ high-J config.

- They are near degenerate (separations: ~100-200 keV)

There may be chiral band-pair among them





Possible multiple chiral doublets in ¹⁰⁴Rh (RELIMINARY)



Possible multiple chiral doublets in ¹⁰⁴Rh

PHYSICAL REVIEW C 88, 024327 (2013) 2.0 (a) Protons in q_{9/2}-shell and neutrons in h_{11/2}-shell Possible presence and properties of multi-chiral-pair bands in odd-odd nuclei $\gamma = 20^{\circ}$ 1.5 with the same intrinsic configuration b1 Ikuko Hamamoto 1.0 b2 Riken Nishina Center, Wako, Saitama 351-0198, Japan b3 and Division of Mathematical Physics, Lund Institute of Technology at the University of Lund, Lund, Sweden 0.5 (Received 26 June 2013; published 30 August 2013) E/K 0.0 PRM calculations: "When the chiral condition is almost fulfilled, the energy distance between the first -0.5 pair and the second pair for the same angular $\lambda_{\rm p}/\kappa = +1.30$ -1.0 $/\kappa = -1.40$ momentum is typically (an order of 1 MeV) several $\Delta_p / \kappa = 0.10$ times larger compared with the splitting of the two $\Delta_n / \kappa = 0.10$ -1.5 (almost degenerate) levels belonging to a given -2.0 chiral pair band." 15 20 25 10 spin I 3.5 (a) Protons in g_{9/2}-shell and neutrons in h_{11/2}-shell 6000 3.0 $\Upsilon = 30^{\circ}$ 5000 🗕 b1 2.5ə b2 - b3 2.0 4000 · ____ b4 1.5 E [keV] E/ĸ 1.0 ● ● Band 1 0.5 Band 2 2000 Band 3 $\lambda_{\rm p}/\kappa = +1.30$ Band 4 0.0 $\lambda_n / \kappa = -1.40$ Band 5 $\Delta_p / \kappa = 0.10$ 1000 -0.5 $\Delta_n / \kappa = 0.10$ -1.0 12 10 14 18 16 20 10 15 25 30

Bands 1,2,3,4 do not correspond to M χ D based on the same configuration

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spin I

CDFT and TAC-CDFT calculations (PRELIMINARY)

CDFT



State	$E_{\mathbf{x}}$	(eta,γ)	Unpaired configuration	π
А	0.00	$(0.27, 23.2^{\circ})$	$\pi(1g_{9/2})^{-1} \otimes \nu(1h_{11/2})^1$	_
В	0.08	$(0.24, 21.4^{\circ})$	$\pi(1g_{9/2})^{-1} \otimes \nu(1g_{7/2})^{-1}$	+
\mathbf{C}	0.29	$(0.22, 13.6^{\circ})$	$\pi(2p_{1/2})^1 \otimes \nu(1g_{7/2})^{-1}$	-
D	0.87	$(0.19, \ 0.0^{\circ})$	$\pi(1g_{9/2})^1\otimes \nu(1g_{7/2})^{-1}$	+
Е	1.21	$(0.36, 8.5^{\circ})$	$\pi(1g_{7/2})^1\otimes \nu(1h_{11/2})^1$	-
\mathbf{F}	2.41	$(0.46, \ 4.2^{\circ})$	$\pi(2p_{3/2})^{-1}\otimes\nu(1g_{9/2})^{-1}$	_
а	1.30	$(0.26, 18.9^{\circ})$	$\pi(1g_{9/2})^{-1} \otimes \nu(1g_{7/2}, 2d_{5/2})^2(1h_{11/2})^1$	_

TAC-CDFT



High-J orbitals were fixed during the calculation

The "effective" configuration of the remaining 8 neutrons can be described as $v(1g_{7/2}, 2d_{5/2})^2$

Good agreement with bands 2,3 and with band 1 at higher spins

PRM calculations (PRELIMINARY)



Possible $M\chi D$ with the two above configurations

Summary

Evidence for transverse wobbling in ¹⁰⁵Pd has been found

- first observation of wobbling in the A~100 region
- first observation of wobbling band based on one-neutron configuraton

Existence of M χ D is proved or possible in several nuclides of the A~100 region

- hints of $M\chi D$ in $^{\rm 105}Rh$
- proved $M\chi D$ based on the same configuration in $^{\rm 103}Rh$
- possible $M\chi D$ in $^{\rm 104}Rh$

Conclusion

Wobbling and multiple chiral bands exist in the A~100 mass region in agreement with the predicted triaxiality

An important thing that we have learnt:

Do not throw away the old experimental data, they can be useful in the future!